

*Portland Experiments on the Flow of Oil in Tubes.*

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(Communicated by Prof. W. McF. Orr, F.R.S. Received May 9,—In revised form June 7,—Read June 13, 1912.)

*Experiments on Water.*

It is a well-known experimental fact that, in the case of water flowing in a straight circular pipe of uniform bore, the resistance per unit length is exactly proportional to the mean velocity if this is below a certain limit, which depends on the radius.

It is also well known experimentally that if the mean velocity exceeds a certain limit, the resistance is approximately proportional to a higher power of the velocity. The index of this power in the case of water has been found to lie between 1.7 and 2. This is usually expressed by stating that the resistance is nearly proportional to the square of the velocity. It will be convenient to refer to these cases as flow according to the first and according to the second law respectively.

When the first law obtains, the motion is in straight lines; but when the flow is subject to the second law, the motion is turbulent. Experiment in the case of water also shows that, between the upper limit of velocity for the first law and the lower limit for the second law, there is a "gap" where neither law obtains. When the velocity corresponds to that in any part of the gap, the motion may be at one time steady and at another time turbulent. The mean velocity corresponding to that of the lower limit of the gap is called the lower critical velocity.

In the case of water flowing in straight circular pipes of uniform bore, the relation between the velocity and resistance at a particular place can be represented as follows:—

$$\text{For the first law ..... } \bar{v} = \frac{g}{32} \cdot \frac{D^2}{\mu'} \cdot \frac{H}{L} = C_1 \frac{D^2 H}{\mu_R L}, \quad (1)$$

$$\text{For the second law ... } (\bar{v})^m = C_2 \frac{D^n H}{(\mu_R)^s L}, \quad (2)$$

where  $C_1$  and  $C_2$  are constants and  $\mu_R$  is proportional to  $\mu'$ .

With regard to fluids flowing in pipes, a matter of vital importance has been the determination of a criterion as to whether the first or second law should hold in a particular case. The principal experimenters in this connection have been Prof. Osborne Reynolds, M. Couette, and Mr. Mallock.

The general result obtained by Prof. Reynolds was that the critical velocity varies directly as the kinematic viscosity and inversely as the radius of the pipe.

The chief theoretical workers in the same field have been Lord Rayleigh, Lord Kelvin, Prof. Osborne Reynolds, and Prof. Orr.\* The analysis, even for the simplest case, is extremely difficult, but the general result above mentioned, as found by experiment, is confirmed.

The last-named writer has treated the whole question of the stability of flow of both viscous and non-viscous fluids in great detail, and has taken into account much that was formerly overlooked by other writers. He has obtained "criteria of stability" for various cases of flow; for a circular pipe his result is

$$D\bar{v}/\mu' = 180. \quad (3)$$

The sense in which his "criterion of stability" is to be interpreted will perhaps be best understood from the following quotation:—†

"It is claimed that in each case the numbers I have found are true least values (but with some reservation as to the effect of end-conditions); that below them every disturbance must automatically decrease, and that above them it is possible to prescribe a disturbance which will increase for a time.

"The numbers obtained . . . give velocities very much below those at which observers have found motions actually to become unstable; this is to be expected."

In connection with the latter clause it is interesting to compare the various results obtained to date; these are as follows:—

Reynolds obtained experimentally the value	1900
Couette        "        "        "        "	2150
Sharp        "        theoretically        "	470
Sharp's value, 470, if corrected for an alleged numerical error, as is done by Prof. Orr, is reduced to	363
Orr obtained theoretically	180

These experimental results were obtained with water, which, as is well known, has a very low viscosity.

No experiments on any fluid of high viscosity flowing in pipes have so far been published, and the experiments about to be described, carried out with Texas oil, would seem to some extent to supply this omission. It should at once be stated that these experiments, while going far, in the writer's view,

\* 'Roy. Irish Acad. Proc.,' 1907, vol. 27, sect. A, Nos. 2 and 3. No references are given to other writers, as these are very fully dealt with in Prof. Orr's papers here quoted.

† *Loc. cit.*, p. 77.

towards proving the existence of a criterion in the neighbourhood of that found by Prof. Orr, were undertaken for a different purpose. They were carried out at Portland, under the entire charge of Mr. G. P. Hayes, B.A., B.E., M.Inst.C.E., and Engineer J. B. Huddy, R.N. (now deceased).

Five different sizes of pipes were used, respectively, 2, 2·875, 3·125, 6, and 10 inches in diameter. The 2-inch and 3·125-inch pipes were of wrought iron, galvanised internally. The 2·875-inch and 6-inch pipes were of cast iron. The 10-inch pipes were formed of lap-welded steel tubes, jointed together with American taper threads and collars.

The ends of the 10-inch tubes did not in this instance come close together, but were separated by an average space of 6 inches, the collar itself forming the tube for this space. The internal diameter of the collar was 10·50 inches, and the average distance from collar to collar was about 15 feet.

The joints in the case of the cast-iron pipes were formed of lead and the ends of the tubes touched in the ordinary manner. The joints in the case of the galvanised pipes were formed with screwed ends and sockets, but were not specially prepared with a view to give perfectly flush joints internally.

The actual diameters of the pipes used were measured near the ends in a number of places with ordinary callipers and the mean result taken. In no case could the error in measuring the diameters be greater than 1 per cent. in the case of the smaller pipes, or more than  $\frac{1}{2}$  of 1 per cent. in the case of the 10-inch pipes. Except in the experiments with the 10-inch pipes, three tanks were used; these were:—

A storage tank of about 2000 gallons capacity, containing a coil of pipes for passing steam, connected to a crane boiler close at hand. This tank was provided with an outlet having a regulating valve attached. The second tank was comparatively shallow and in the form of a tray, and was placed under the outlet from the storage tank.

The third tank of 600 gallons capacity was carefully calibrated at intervals, each of which corresponded to 50 gallons capacity. The inlet end of the pipe experimented on, projected into the side of the second tank and its outlet end was placed directly over the third tank. The outlet of this pipe was supplied with a short bend turned in the upward direction, and in no case was this end of the pipe submerged.

The oil experimented on was placed in the storage tank and raised to the requisite temperature by means of steam passed through the coil placed within the tank. The temperature of the oil in the storage tank was kept constant and uniform by regulating the supply of steam, and also by continuous stirring of the oil. An attendant who had charge of the valve attached to the storage tank regulated the supply so as to keep the oil at all times up to a

mark on the sides of the tray and thus insure a constant head over the intake end of the pipe. The temperature of the oil was measured both at the intake and at the outlet of the pipe by means of ordinary service thermometers and in each case the readings were compared with those of a standard instrument in which the errors were known.

In the case of the experiments with the 10-inch pipe the oil was merely allowed to flow by gravitation from the large storage tanks on the hillside to tank steamers in the harbour. The oil was not heated, but the temperature of the oil was taken in the storage tanks and also in the tank steamers.

In all cases the head was measured by the difference of level between the surface of the oil in the tray and the centre of the pipe at the outlet. This was done with a 14-inch dumpy level and the maximum error in any case except that of the 10-inch pipes probably did not exceed 0.01 foot. In the case of the 10-inch pipe the error in the measurement of the head after making ample allowances was almost certainly less than 1 foot.

The tanks and pipes were arranged as shown on the accompanying sketch (fig. 1).

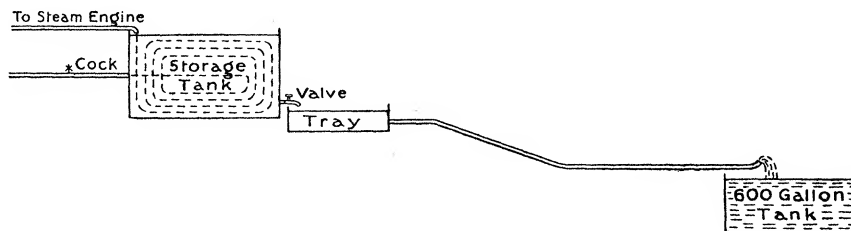


FIG. 1.

The oil experimented on was from Texas; its specific gravity was 0.9375 at 60° F. The variation of density with temperature was not noted in this instance as it had been found to be very slight, but for purposes of calculation the density has been taken as given by the mean formula:—

$$\rho_T = \rho_0 - 0.000362 T. \quad (4)$$

The viscosity of the oil was measured with a Redwood's viscometer at intervals of 5° F., for all temperatures between 35° and 90°.

In the case of the oil on which the Portland experiments were performed the times for the flow of a given volume were plotted as ordinates and the temperatures as abscissæ and a mean curve was drawn. The observations were then trimmed so as to make them conform as nearly as possible with the mean curve. The logarithms of the trimmed values were then plotted in the same way, and it was found that these almost fell on a straight line.

Redwood's viscosity at any temperature was found to be given by the formula

$$\log \mu_R^T = \log \mu_R^{60} + 0.02053 (60 - T) + 0.00042 (60 - T)^2. \quad (5)$$

It appears from Poiseuille's experiments that in the case of water  $\mu'$  varies as

$$[1 + 0.03368 T + 0.00221 T^2]^{-1}. * \quad (6)$$

Also from the experiments of Helmholtz and Piotrowski,

$$\mu = 0.014061 \text{ at } 24.5^\circ \text{ C.} \quad (7)$$

*Criterion of Stability.*

If we wish to obtain a criterion as to whether the first or second law should hold in a particular case and be applicable to a fluid for which Redwood's viscosity is known, we have the equations (1).

From these, if it is remembered that  $C_1$  has been calculated on the assumption that  $D$  is expressed in inches, there is easily obtained the relation

$$\mu_R/\mu' = 144 C_1, \quad (g = 32). \quad (8)$$

The mean value of  $\log C_1$ , as found from Experiments 7 to 19 inclusive (omitting Experiment 16, which applies to a solitary length of pipe and gives a somewhat discrepant result), is 3.4780.

$$\text{There is thus obtained } \mu_R/\mu' = 432,900. \quad (9)$$

Taking now Prof. Orr's criterion of stability,

$$\begin{aligned} D\bar{v}/\mu' &= 180 \text{ where } D \text{ is expressed in feet,} \\ &= 2160 \quad \quad \quad \text{,,} \quad \quad \quad \text{,,} \quad \quad \quad \text{inches,} \end{aligned}$$

there is obtained from equation (9) the criterion

$$D\bar{v}/\mu_R = 0.005 \text{ (say).} \quad (10)$$

In order to see how far this is confirmed by the present experiments, each set of tests has been arranged according to lengths and diameters of pipes and plotted in a separate diagram, the values of  $\log \bar{v}$  being plotted as ordinates and those of  $\log H/\mu_R L$  as abscissæ. The plottings are shown in figs. 3, 4, and 5.

Consideration of equations (1) shows that so long as they hold and are applied to a single diameter of pipe the results should plot in a straight line inclined at  $45^\circ$  to the axes, any decided declension of the line from  $45^\circ$  indicating a failure of the capillary law.

Again, consideration of equation (2) shows that unless  $s$  is unity the

\* 'Phil. Trans.,' 1883, p. 943.

points as plotted in the above scheme need not necessarily fall on a straight line beyond the point where equations (1) break down. When, however,  $s$  is unity the points should still fall on a straight line inclined at the angle  $\tan^{-1} m$  to the  $\bar{v}$  axis. The point of intersection of the two branches, where such exist, has been taken as corresponding to a practical critical value.

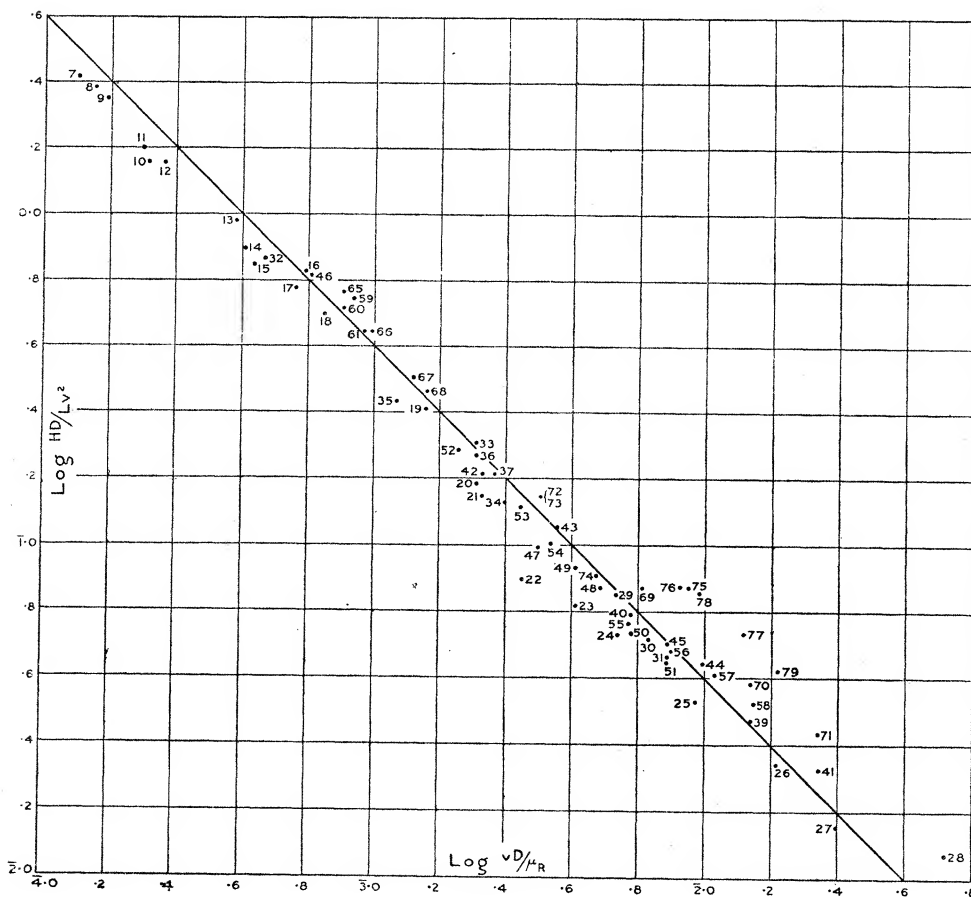


FIG. 2.

It will be noticed that there are eight diagrams and in the first of these (fig. 2) the results of the whole of the experiments for all the pipes are plotted to the co-ordinates  $\log \frac{\bar{v}D}{\mu_R}$  and  $\log \frac{HD}{L\bar{v}^2}$ ; this diagram has been constructed entirely at the suggestion of Prof. Orr.

On referring to the other diagrams (figs. 3, 4, and 5) it will be seen at once that the only cases throughout the whole of the experiments in which

two distinct branches occur are in two sets of experiments on the 6-inch pipe, plotted on fig. 5, viz., Experiments 52 to 61 and Experiments 65 to 71.

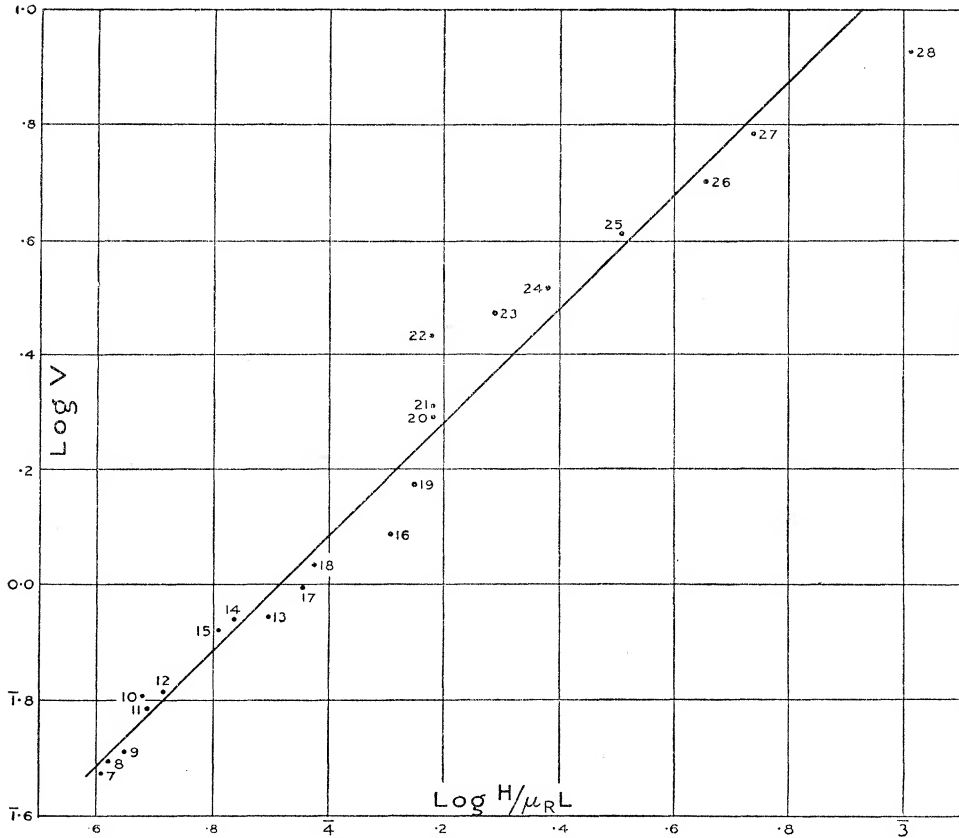


FIG. 3.—2-inch galvanised pipe.

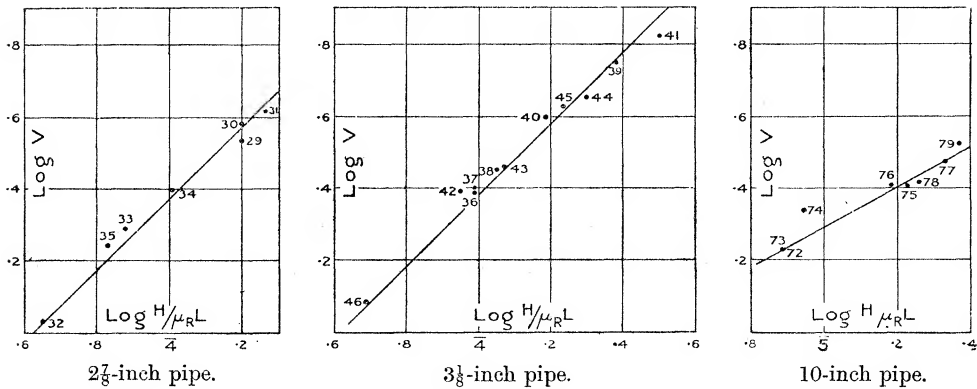


FIG. 4.

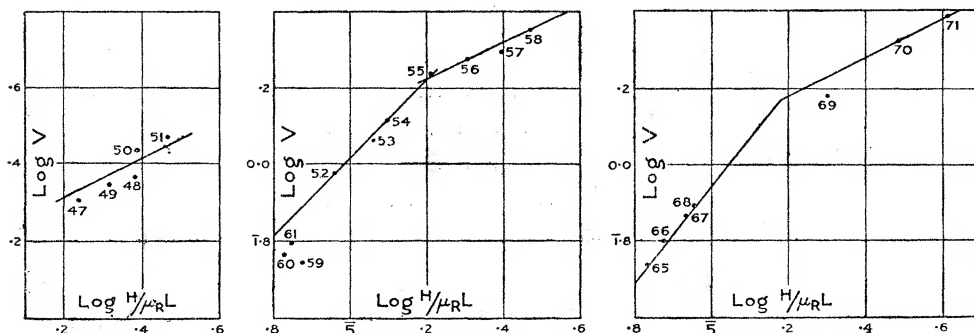


FIG. 5.—6-inch pipe.

These two sets of experiments have been separately plotted for the reason that any errors in the measurement of the head producing flow, or errors in the diameters of the pipes caused in manufacture, or due to imperfect jointing, or errors due to bends, etc., must of necessity be constant throughout each set, while these errors may differ to an appreciable extent as compared between one set and the other.

The value of  $D\bar{v}/\mu_R$  at the point of intersection of the two branches can be calculated from the identity

$$\frac{D\bar{v}}{\mu_R} = \bar{v} \frac{H}{\mu_R L} \cdot \frac{DL}{H}, \quad (11)$$

seeing that  $L$  and  $D$  are known constants and  $H$  is very nearly constant and can be obtained with considerable accuracy at the point of intersection from the value of  $\log \bar{v}$  as scaled at that point.

In the case of the two sets of experiments referred to the values corresponding to the critical velocity are as follows:—

Experiments.	Value of $H$ .	Value of $D\bar{v}/\mu_R$ .
52 to 61	3.04	0.00565
65 to 71	2.70	0.00503
Mean = 0.00534.		(12)

The values as found by experiment are thus almost identical with that obtained by Prof. Orr on theoretical grounds; the difference in the case of the higher experimental result is well within the limits of experimental errors or indeed errors in plotting.

All the experiments in which  $D\bar{v}/\mu_R$  is equal to or greater than 0.006 have been selected, and an attempt has been made to obtain values for  $m$ ,  $n$ , and  $s$  in equation (2) to fit these experiments. This has been attended with only partial success. The values which best conform to the whole of those experiments appear to be:—

$$m = 1.50, \quad n = 1.20, \quad \text{and} \quad s = 0.75.$$

How far these values succeed may to a slight extent be gauged by the uniformity of the values of  $\log C_2$  obtained. These were as follows:—

2-inch galvanised pipe .....	$\log C_2 = 3.3290$
$2\frac{7}{8}$ -inch cast-iron pipe .....	„ = 3.2868
$3\frac{1}{8}$ -inch galvanised pipe .....	„ = 3.3336
6-inch cast-iron pipe .....	„ = 3.3697
10-inch steel tubes .....	„ = 3.3242

It should, however, be remarked that the values of  $\log C_2$  differ from one another in any one set of experiments to a much greater extent than is here shown, and thus the derived formula

$$(\bar{v})^{1.50} = C_2 \frac{D^{1.20}}{\mu_R^{0.75}} \cdot \frac{H}{L}, \quad (13)$$

applicable to velocities within the range of the experiments above the critical point, would not appear to be at all well established. It can only be looked on as perhaps the best mean result. Indeed other relations can be found which fit the greater portion of the experiments very fairly.

With regard to equations (2) and (13), the reader may recollect that Prof. Osborne Reynolds put forward on theoretical grounds an equation of the type

$$\frac{H}{L} = \frac{v^2}{D} f\left(\frac{vD}{\mu'}\right), \quad (14)$$

which includes the particular case of (2), in which  $m = 3 - n$ ,  $s = n - 1$ , and considered that his experimental results for velocities higher than the critical are satisfactorily represented by this particular case if  $n$  has the value 1.723. Prof. Orr has indicated to the writer, however, that the theory on which (14) is based supposes that the motions in pipes of different diameters are dynamically similar; this implies that the eddy-systems are similar and on linear scales proportional to the diameters; it seems at least open to question whether this assumption is correct.

Prof. Orr has also indicated to the writer that he inclines to the view that his so-called “criterion of stability” obtained by the method which Reynolds was the first to employ does not by any means indicate a sharp dividing line between stability and instability, and that when the velocity exceeds or even approaches that given by the criterion, it is not unlikely that it depends on circumstances which may be described as accidental whether the motion is stable or unstable, as also what the value of the resistance may be.

He has also pointed out that the critical velocity below which every small disturbance automatically decays (which is the velocity given by his

criterion) and the critical velocity (if there be one) beyond which the first law of resistance fails have not the same meaning and may well have different values in actual fact.

In conclusion, it should be stated that the results of these experiments are published by the courtesy of Colonel Sir Edward Raban, K.C.B., R.E., Director of Works of the Navy.

*Borohydrates.*—Part I.

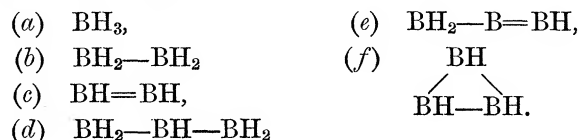
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(Received May 20,—Read June 27, 1912.)

*Introduction.*

In 1881 Jones and Taylor\* demonstrated the existence of a compound of boron and hydrogen, and 20 years later Ramsay and Hatfield† published a preliminary account of a research on the hydrides of boron, describing experiments which, even if they cannot be regarded as conclusive, opened up a field of investigation well worthy of attention. They found that on passing the gas evolved by the action of dilute acids on magnesium boride through a bulb cooled in liquid air, a solid substance was deposited which, on warming the bulb, volatilised and could be collected as a gas. From the results of the analysis of the gas, and its density, its formula appeared to be  $B_3H_3$ ; but it appeared to consist of two isomeric substances, one stable, and the other unstable and readily decomposed by reagents. The gas which was not condensed by liquid air appeared to consist mainly of hydrogen, but to contain a hydride or hydrides of boron.

Assuming the trivalency of boron, it is possible, as Ramsay and Hatfield point out, that a very large number of “hydroborons” may exist. The simpler of these may be represented by the formulæ:—



\* ‘Chem. Soc. Trans.,’ vol. 39, p. 213.

† ‘Chem. Soc. Proc.,’ vol. 17, p. 152.